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Modelling the Qualitative Structure of Scots Pine Trunks Using a Random Process

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Abstract. The study highlights a method for predicting the volume of qualitative categories of trunk wood for timber trunks of Scots pine, using a model of distribution of various qualitative zones along the trunk. The study is based on empirical material collected in the main-use cutting area, where the length of various qualitative wood zones from the butt to the top was determined for 245 model trees. The paper uses a semi-Markov probabilistic model to generalise the order of appearance and length of zones of timber wood, firewood, and waste, which was determined by three parameters: 1) input probabilities of the appearance of the corresponding qualitative zone in the butt part of the trunk (initial state); 2) matrix of distribution of zone lengths; 3) matrix of probabilities of changes in qualitative zones at different heights of the trunk. According to research data, it is generally accepted that pine trunks begin with timber wood. The beta distribution function is used to model the length of qualitative trunk zones, the parameters of which are selected depending on the relative height of the beginning of the corresponding trunk zone. The probabilities of changes in qualitative zones are calculated based on empirical data. The study identified that the distribution of timber and firewood lengths depends on the absolute height of the trunk location and the height of the trunk. For a mathematical generalisation of this process, the paper defines four zones within which the distribution of the length of the timber part of the trunk can be described by a single function. The probabilities of changes in qualitative zones are modelled depending on the relative height of trunks. On this basis, the method of simulation modelling of initial data sets has been developed, which can be used to develop tables of dimensional and qualitative wood structure for trunks of different diameters, heights, and categories of technical suitability. The study applies only to timber pine trunks, so other patterns are probable for trunks of other tree species, semi-timber, and firewood trunks. The developed methodology is appropriate to use when updating the tables of trunk volume distribution by variable-quality categories, which now need to be updated by introducing new requirements for the classification of timber wood in Ukraine

Keywords: timber tree, distribution function, semi-Markov probability model, tables of dimensional and qualitative structure

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Introduction

The transition of the Ukrainian forest industry to European standards for the classification of round timber by size and quality has led to the need to update the regulatory framework used in production when accounting for the trunk stock of wood. During 2019-2020, the first stage of development of standards was conducted, which provided for modelling the yield of round timber by thickness classes for ten main tree species: pine, spruce, fir, oak, beech, ash, hornbeam, birch, alder, aspen (Bilous et al., 2020). Notably, the proposed standards allow performing a preliminary assessment of the maximum possible yield of timber wood and its dimensional structure. This is due to the use of research material available at the Department of Forest Mensuration and Forest Management of the National University of Life and Environmental Sciences of Ukraine (about 20 thousand model trees), which was collected over the past two decades according to already abolished interstate standards. Therefore, to improve these standards, it is necessary to attract new empirical data. Evidently, getting a sufficient number of model trees will require substantial financial costs, and most importantly – time.

Most of the standards in Ukraine are developed by equalising empirical data using regression analysis methods. It is necessary to have a substantial amount of research material – from 400 to 600 model trees for each tree species to obtain such models with a sufficient level of adequacy and accuracy (Nikitin & Shvidenko, 1972). Because of this, there is a need for a deeper study of the relationship of random variables that affect the formation of the dimensional-qualitative structure of the volume of a tree trunk. In particular, the application of random process theory and simulation modelling allows obtaining a sufficient number of implementations of the simulated process to ensure the established accuracy and level of confidence probability.

In Ukraine, to display the dimensional and qualitative structure of trunks, the random process was first used in 1983 when performing a study on optimising the structure of logging in the conditions of the Ukrainian Carpathians (Developing Scientific Background, 1983). The idea of application belongs to Ya.A. Yudytsky,

who published a methodological basis for the possibility of applying random functions in solving sorting problems in 1985 (Yudytsky, 1985). This method was further developed in the papers of A.V. Poliakov, in particular, in the scientific finding “Adaptive industrial sorting of the cutting fund” (Poliakov & Poliakov, 1999, 2008, 2009). This was used to compile some standards (Kashpor & Strochynskyi, 2013) and update them in 2020 according to new European standards (Bilous et al., 2020). These studies are based on a stochastic model of the distribution of qualitative zones along the trunk, developed based on the theory of Markov processes for the assumption that the length of zones (i.e., the time when the system is in a certain state) is distributed according to the exponential law (Koroliuk & Turbin, 1976). As a result, it is established that the quality class of the next trunk zone, as an analogue of the next state of the Markov system, depends only on the quality class of the current zone (the current state of the system) and does not depend on its earlier states.

Modelling the probabilistic distribution of the appearance of the corresponding qualitative zone along the trunk is a difficult task even from the standpoint of the possibility of using statistical methods. The distribution of response values, i.e. the volume of wood of the corresponding quality class in the trunk stock, is usually far from normal. Because of this, based on trunk matching models (Adamec et al., 2019) and timber wood volume models (Fonweban et al., 2012) it is possible to successfully predict its volume in a separate trunk, but this approach does not work at the level of forest stands.

The problem posed requires a more complex methodological solution that would allow considering the probability of the appearance of a particular quality zone on the trunk, and then – the distribution of volume by quality classes. According to this, Fortin et al. (2009), considering each qualitative zone of the trunk as a binary characteristic (the presence of which on the trunk can be characterised by a value of 1, and the absence – 0), at the first stage, linear logistic regression was used to model the probability of occurrence of these zones depending on the biometric indicators of the tree trunk. Only

then a model of wood volume distribution of the corresponding quality class for birch and maple tree trunks in the province of Quebec in Canada were considered. Notably, the models were developed separately for four quality categories (categories of technical suitability) of trunks.

A similar approach, based on binary logistic regression to predict the probability of the appearance of qualitative zones on the trunk, was later used by specialists of the US Forest Service (Banzhaf et al., 2016). According to the researchers, a sufficient level of accuracy of such models should be expected only for general indicators aggregated at the level of stands. In some cases, modelling of the qualitative structure of the trunk volume was considered from the standpoint of the theory of statistical classification of trunk zones by quality. For example, Schneider et al. (2008) combined polynomial logistic regression to predict the number of finite wood quality classes and binomial logistic models to determine the probability of occurrence of each of them for trunks.

In general, the scientific literature in the context of the presented study is quite limited. Despite the availability of separate publications on predicting the qualitative characteristics of stem wood, the possibility of simultaneous modelling of dimensional and qualitative indicators was almost not considered. Based on this, it is necessary to recognise the importance of scientific ideas proposed earlier by Ya.A. Yudytsky

and O.V. Poliakov (Poliakov & Poliakov, 1999, 2008, 2008; Yudytsky, 1985).

The purpose of the study is to substantiate based on research material a model of the random process of the length of trunk zones by quality, namely timber wood, firewood, and waste. Determine the possibility of the practical application of the obtained process model in the development of standards.

Materials and Methods

Research material.

The study used research material collected in the SE “Horodnyansky lishosp” of the Chernihiv region. On the laid test area, 245 timber model scots pine trees were cut down, for which the height of the stump was determined, the quality zones were evaluated on the trunks, and their length was measured. A total of 821 timber zones and 307 firewood zones were allocated for model trees. The minimum length of timber was assumed to be 1.0 m, while for firewood segments – 0.5 m. After that, the employees of the company bucked the trunks for timber according to the current specifications and accepted the harvested forest products. A total of 266.7 m³ logging products is taken from the coup, including 248.5 m³ timber wood and 18.2 m³ firewood.

The general characteristics of the research material are presented in Table 1, where the height of the trees is given including the height of the stump.

Table 1. Statistical characteristics of model trees

Statistics	Diameter at a height of 1.3 m, cm	Height, m	Length of timber wood	
			m	share of height, %
Average value	36.1	27.1	19.5	71.6
Minimum value	20	18.7	7.1	27.7
Maximum value	53	37.4	27.1	88.6
Standard deviation, (σ)	7.0	2.6	12.4	1.0
Coefficient of variation (v), %	19.4	9.6	63.6	1.4

When analysing statistics, it is necessary to pay attention to a fairly wide range of variability in mensuration indicators of empirical data. This is indicated by the coefficients of variation of the diameter (19.4%) and the length of timber wood in absolute terms (63.6%). Instead, the relative values of the length of a timber tree are quite tightly grouped around the average (71.6%), with a standard deviation of only 1%.

Modelling the distribution of qualitative trunk zones.

The random process model determines the order of appearance and length of trunk sections of a certain quality from the butt to the top. In general, a quality class can have seven discrete values: *A, B, C, D* – for timber wood; *E* – for firewood areas that are limited on both sides by timber areas; *F* – for firewood, the zone of which begins after the last timber zone; *G* – waste (top of the trunk with a diameter at the lower end <2 cm). Notably, firewood zones of the *E* class were identified in a small number of model trees (out of 307 firewood zones, only 64 are classified as *E*), they had an average length of 1.2 m. Assuming that the effect of such segments on the total volume distribution is insubstantial, they were not considered in the simulation.

In accordance with the purpose of the study, the task can be simplified so that only one class of timber wood quality is considered – $T \in \{A, B, C, D\}$ and firewood – *F*. Since there are no transitions from the latter to the timber one, it can be assumed that *F* – is the absorbing (irrevocable) state of the system. In addition, placing a transition point from the zone on trunk *F* to zone *G* is not probabilistic in nature and depends only on the coincidence of a particular trunk. Given this, the qualitative structure of the trunk can be described as a random process where the quality class of the zone (may matter *T* and *F*) is the state of the process, and the length of the zone is the time it stays in this state. That is, the process states change discretely, and the time continuously – from the minimum (for timber wood – 1.0 m, for firewood – 0.5 m) to a certain maximum value.

Depending on the law of distribution of time being in a certain state, processes with discrete states and continuous time can be considered as Markov or semi-Markov processes. Semi-Markov

processes are a direct generalisation of Markov chains and processes and can be structurally defined in several ways (Koroliuk & Turbin, 1976).

In this paper, a semi-Markov model was chosen, which is defined by: 1) the initial distribution of input probabilities; 2) the matrix of functions of the distribution of random residence time in the state *i* provided that the transition takes place in the state *j*; 3) the transition probability matrix of the Markov chain. This approach allows assuming that the distribution functions of random residence time in the state *i* do not depend on the following state *j*, which the system will shift to. In this case, the distribution functions are not related to the transition probability matrix and can only depend on time and/or other indicators of the system. In addition, it is also possible to consider changing states *ii* by taking them into account in the transition matrix. In other words, the length of a zone does not depend on the quality of the next zone, and transitions can be made to the same quality class (from *T* to *T*).

With this design of the process, its evolution can be described as follows: starting from a certain state *i*, which is defined by a vector *p* in the initial distribution, the system spends time in this state ζ_i with distribution function $G_i(x)$; after the ζ_i time has elapsed, the state changes to *j* with a probability of p_{ij} , which is defined by the transition matrix *P*; once in the state *j*, the system spends in this state ζ_j time, etc., until the process goes into an absorbing state.

Process modelling methodology:

1. The initial distribution of input probabilities was calculated based on empirical data. In general, it is a vector of relative frequencies of occurrence of a certain quality class at the beginning of the cut trunk (from the stump):

$$p = \{p_i, i \in (1, 2, \dots, n)\}.$$

If quality classes that can acquire 5 attribute values (*A, B, C, D, and E*), provide numeric values, then $n=5$, and a random variable $x_i \in (1, 2, \dots, 5)$. The distribution function of such a discrete series will have the form:

$$F(x) = \sum_{i: x_i < x} p_i, \quad (1)$$

where $F(x)$ – distribution function; x – function argument; x_i – value of a random variable; p_i – probability of occurrence i -th quality class; i – summation indexes for which the inequality is met $x_i < x$.

Accepting the simplification that a timber tree is one class (T), then $n=2$, and a random variable $x_i \in (1, 2)$.

When generating a random process, the stump height (h^{st}) can be adopted as a fixed or mathematical model based on biometric indicators of the trunk. Considering the height of the stump is necessary because the vast majority of mathematical models of the generatrix describe the coincidence of the trunk, starting from the zero mark (from the earth's surface).

2. The law of time distribution (i.e., the lengths of timber zones) between the points of state change was determined based on research material. The absolute heights of the start points of each zone are listed as relative, according to the maximum possible length of the timber tree for a specific trunk height and the minimum length of the timber zone using the formula:

$$\hat{h}_i = \frac{h_i - h^{st}}{\tilde{h}_{max}^{zone} - h^{st} - \tilde{l}_{min}^{zone}}, \quad (2)$$

where \hat{h}_i – relative height of the beginning of the zone; h_i – factual height of the beginning of the zone, m; \tilde{h}_{max}^{zone} – mathematical model of the maximum height of the end of a timber tree, m; h^{st} – factual stump height, m; \tilde{l}_{min}^{zone} – minimum length of the timber area, m.

The choice of such a reference is due to the fact that the beginnings of timber zones are located on the trunk in the height range $h_i \in [h^{st}; \tilde{h}_{max}^{zone} - \tilde{l}_{min}^{zone}]$, i.e. above the point $\tilde{h}_{max}^{zone} - \tilde{l}_{min}^{zone}$ the process with probability 1 will move to the zone F of the firewood and will remain there until the transition to the state G.

The maximum height of the end of the timber part of pine trunks was determined based on a linear empirical model:

$$\tilde{h}_{max}^{zone} = a_0 + a_1 \cdot h, \quad (3)$$

where \tilde{h}_{max}^{zone} – maximum height of the end of the timber part, m; h – trunk height, m; a_0, a_1 – equation parameters.

It is necessary to present the values of zone lengths also in relative units to obtain experimental data in a comparable form. They were recalculated using the formula;

$$\hat{l}^{zone} = \frac{l^{zone} - \tilde{l}_{min}^{zone}}{\tilde{l}_{max}^{zone} - \tilde{l}_{min}^{zone}}, \quad (4)$$

where \hat{l}^{zone} – relative length of the zone; l^{zone} – actual zone length, m; \tilde{l}_{min}^{zone} – minimum zone length, m; \tilde{l}_{max}^{zone} – maximum zone length, m.

The numerator of this formula shows the random component of the length, and the denominator shows the range of values, that is, the difference between the possible maximum and minimum length.

The maximum length of the zone is limited to the maximum possible height of the end of the timber wood, so it was calculated using the equation:

$$\tilde{l}_{max}^{zone} = \tilde{h}_{max}^{zone} - h_i, \quad (5)$$

where \tilde{l}_{max}^{zone} – maximum zone length, m; \tilde{h}_{max}^{zone} – mathematical model of the maximum end height of timber wood (3), m; h_i – factual height of the beginning of the zone, m.

Modelling of the length distribution of qualitative zones was performed according to the given relative indicators using the beta distribution function, which describes continuous random variables that have a domain of definition in the interval [0-1]:

$$f(x) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1}, \quad (6)$$

where $B(\alpha, \beta)$ – beta function; x – independent variable, m; α, β – parameters.

The parameters of the distribution function were identified using the "Solution search" package in MS Excel. The optimisation criterion is the minimum sum of the squares of the differences between the empirical data and the model response. Next parameters α and β were aligned with the relative height using an equation of the form:

$$y = a_0 \cdot \exp(a_1 \cdot \hat{h}_i) + a_2, \quad (7)$$

where y – corresponding parameter of equation (6); a_0, a_1, a_2 – parameters; \tilde{h}_i – relative height.

The length of the firewood area (F) can be identified for a specific implementation of a random process as the difference between the height of its end (i.e., the transition to the waste zone G) and the height of the end of the timber part:

$$\tilde{l}^{fw} = \tilde{h}^{fw} - \tilde{h}^{zone}, \quad (8)$$

where \tilde{l}^{fw} – length of the firewood zone, m; \tilde{h}^{fw} – height of the end of the firewood zone, m; \tilde{h}^{zone} – end height of timber wood, m.

Height \tilde{h}^{fw} is calculated by the generative equation, which is covered in [10], for the condition that $f(h^{tr})=2$ cm (minimum diameter of the firewood in the upper section). The height of the end of a timber tree can be identified by the formula:

$$\tilde{h}^{zone} = \tilde{h}^{st} + \sum_1^i \tilde{l}_i^{zone}; \quad (9)$$

where \tilde{h}^{zone} – end height of the timber part, m; \tilde{h}^{st} – stump height, m; i – number of generated timber segments, pcs.; \tilde{l}_i^{zone} – length of the i -th timber segment.

3. Transient probabilities. As mentioned earlier, the zone quality class can have two values: – T and F . Hence, the transition probability matrix will take the form:

$$P = \begin{Bmatrix} p_{TT} & p_{TF} \\ p_{FT} & p_{FF} \end{Bmatrix}, \quad (10)$$

where p_{ij} – probability of transition from the state i to the state j .

Accepting that p_{TF} – is the value of some function $F(x)$, then $p_{TT}=1-F(x)$, since events TT and TF are incompatible and make up a complete group. In addition, the function $F(x)$ is non-decreasing, because moving up the trunk, the probability of switching to firewood wood increases. If F – is an absorbing state, then $p_{FT}=0$, and $p_{FF}=1$, i.e. transitions FT do not happen in the system. Hence, to determine the elements of the first row of the matrix, it is necessary to find $F(x)$. The beta distribution function (6) was also used for this purpose.

Implementation of a random process.

It is assumed that trunk diameter ($d_{1,3}$) and its height (h) an uniform distribution within the degree of thickness (gradation of 4.0 cm) and the height category is accepted, respectively:

$$d_{1,3} \in [d^{st}-2,0; d^{st}+1,9]; \quad (11)$$

$$h \in [h_{min}^p; h_{max}^p], \quad (12)$$

where d^{st} – average diameter of the degree of thickness, cm; h_{min}^p, h_{max}^p – minimum and maximum heights, respectively.

The algorithm for generating a qualitative structure of pine trunks using a random process can be presented in Table 2. The algorithm is implemented by the simulation method using inverse random variable distribution functions, where for each step (except for the third and last two), a uniformly distributed random number is generated as an argument to these functions.

Table 2. Algorithm for generating a high-quality trunk structure

Nº	Indicator	Source	Link
1	$d_{1,3}$	thickness limits	(11)
2	h	height discharge limits	(12)
3	\tilde{h}^{st}	average value or mathematical model	–
4	zone quality class	input probabilities	(1)
5	timber area length	distribution function	(2–7)

Table 2, Continued

Nº	Indicator	Source	Link
6	zone quality class	transient probabilities	(2, 6, 10)
7	timber area length	distribution function	(2-7)
...
$n-1$	length of the firewood area	generating function	[10], (8, 9)
n	waste	remainder to the top	-

Results and Discussion

In the study, it is accepted that the trunk starts with timber wood, that is, the initial state of the process is a quality class T . Under such conditions, the function of the initial distribution of input probabilities (1) for the accepted quality classes of zones (E, T) will acquire values: $P_E=0; P_T=1$, because the appearance of an E zone on the trunk wasn't considered. The height of the stump in

the experimental material lies in a rather narrow range of 0.10-0.15 m, so for modelling a random process, its average value is taken ($\tilde{h}^{st}=0,133$ m).

Fig. 1 shows the dependence of the absolute heights of the endpoints of the timber part of model trees on the trunk height, and a graph of their maximum values according to the mathematical model (3) and the identified parameters: $a_0=-2,896; a_1=0,9851$.

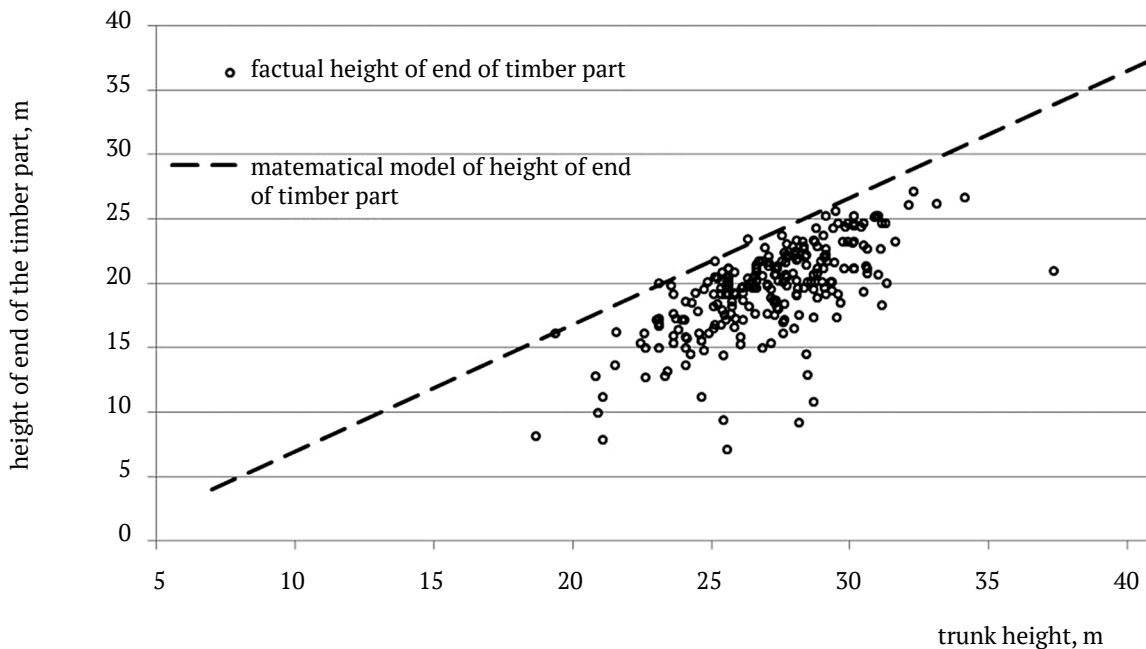


Figure 1. Dependence of the height of the end of the timber part of the trunk on its height

Applying equation (2-5) and considering that the minimum length of the timber zone does not depend on the mensuration indicators of the trunk

and is equal to $\tilde{l}_{min}^{zone}=1$, an array of experimental data on the dependence of the relative length of the zone on the relative height of its beginning is obtained

(Fig. 2). Analysing this graph, it can be assumed that the distribution of the relative length of the zone at an arbitrary relative height depends only on this height and does not depend on the height of the trunk.

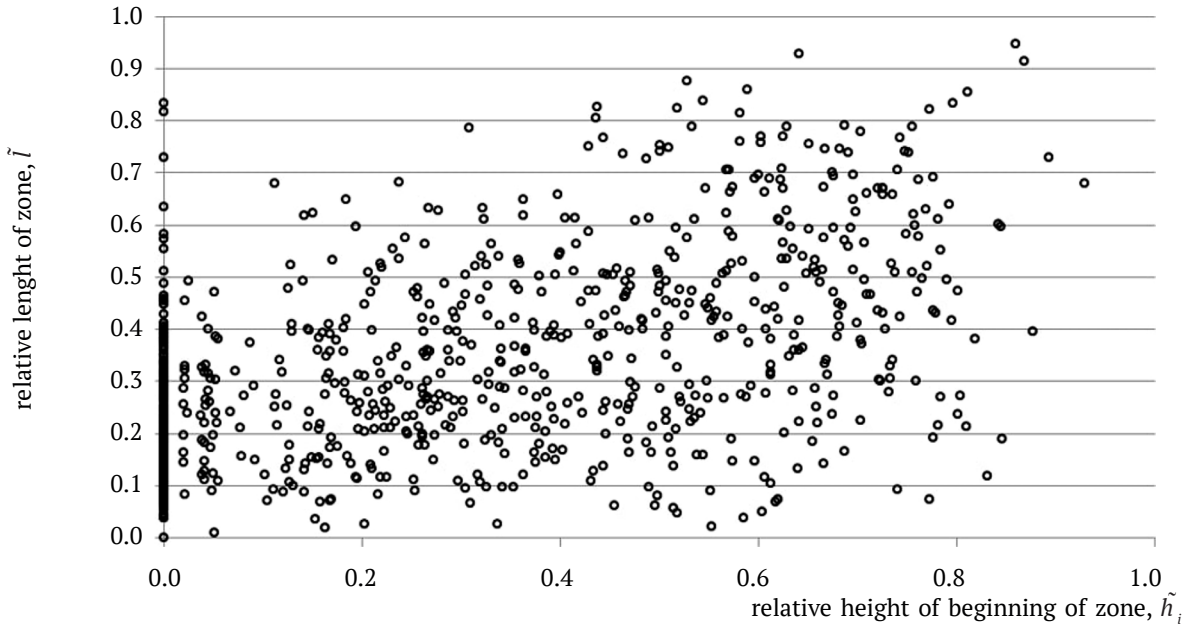


Figure 2. Dependence of the relative lengths of timber wood zones on the relative height of their beginning

Arrays of empirical data were analysed to confirm this assumption for different groups of trunk heights by *F*- Fischer's criterion and Student's *t*-test. This analysis confirmed the hypothesis that the groups belong to the same general population (at the level of substantiality $\alpha=0.05$).

In the future, the relative frequencies of occurrence of zone lengths for gaps in the relative height of the trunk are calculated: [0-0,1], [0,1-0,4], [0,4-0,7] and [0,7-1,0]. Graphs of empirical distribution functions for these intervals are shown in Fig. 3.

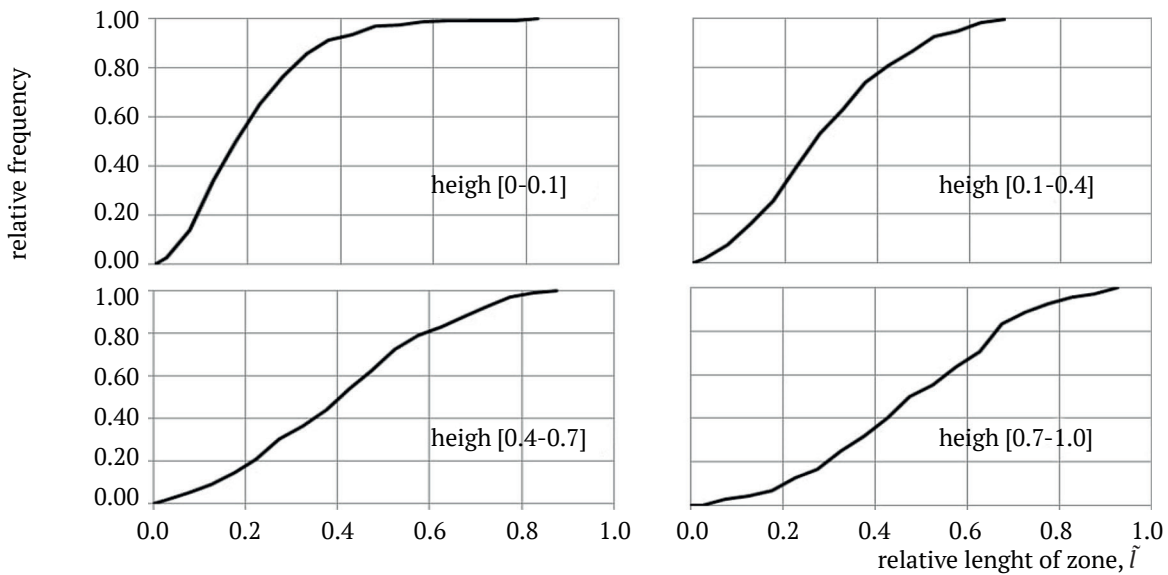


Figure 3. Empirical functions of distribution of relative lengths of timber wood zones at different relative heights of their beginning

From the above graphs, it can be seen that the distributions may differ from the exponential one, with an uncharacteristic form for the latter at the bottom and a bounded domain of definition.

Therefore, a random process can be classified as semi-Markov. Calculated parameters α and β of the equation (6) for those shown in Fig. 3 empirical distributions are shown in Table 3.

Table 3. Algorithm for generating a high-quality trunk structure

Parameter	Relative trunk height intervals			
	[0.0-0.]	[0.1-0.4]	[0.4-0.7]	[0.7-1.0]
α	1.946	2.156	1.997	2.467
β	7.979	5.388	2.874	2.636

The series of parameters that are shown in Table 3 were aligned using the equation (7). Graphs of their simulated values are shown in Figure 4, and the parameters of mathematical models are shown in Table 4.

Therefore, using mathematical models (2-7), at any point on the trunk, a distribution of the absolute length of the zone can be obtained, the parameters of which depend on the absolute height of this point and the height of the trunk (Fig. 5).

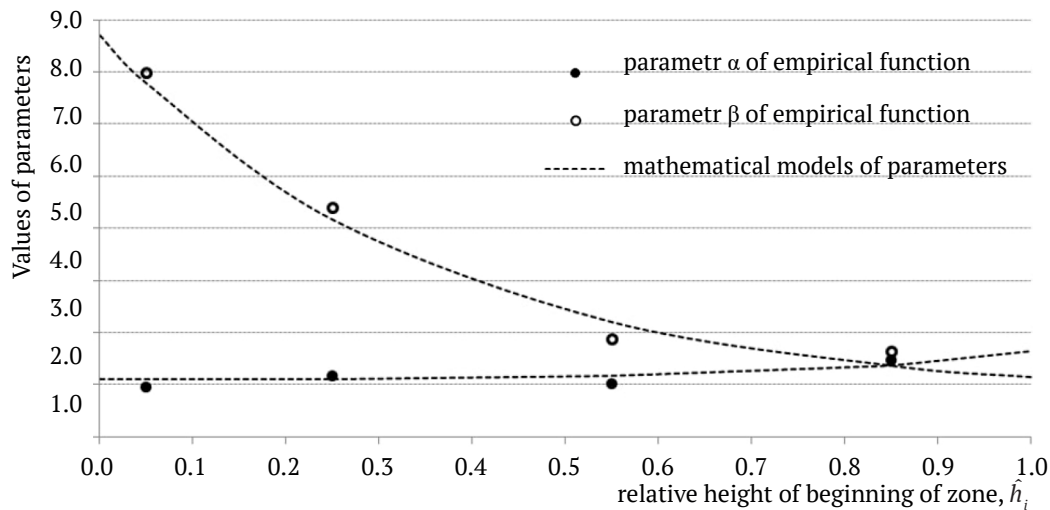


Figure 4. Mathematical models of parameters

Table 4. Parameter values of equation (7)

Parameter of equation (6)	Parameters of equation (7)		
	a_0	a_1	a_2
α	$7.113 \cdot 10^{-5}$	8.395	2.300
β	7.113	-2.846	1.387

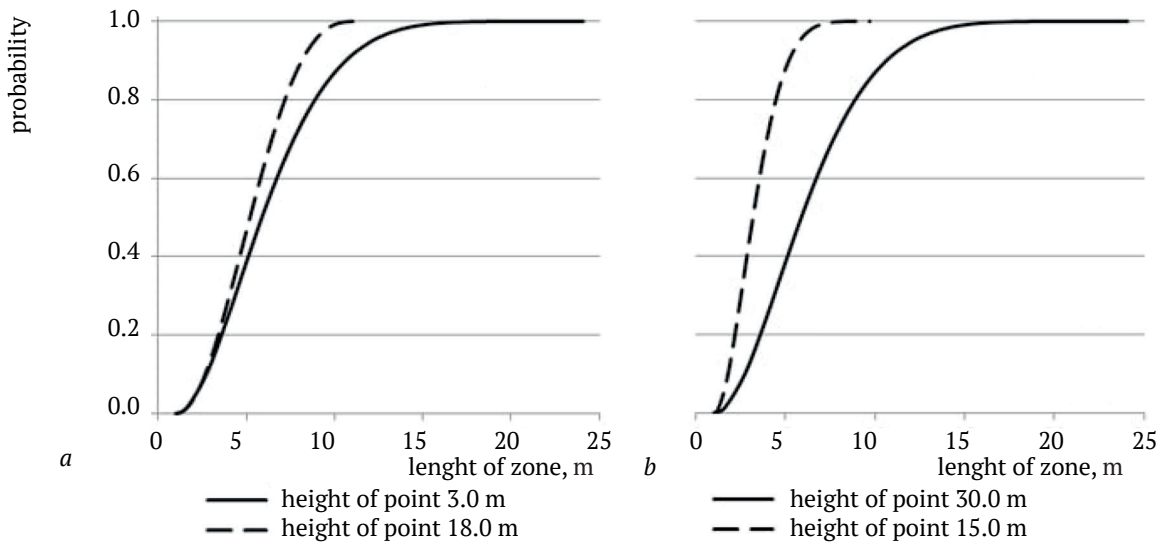


Figure 5. Graphs of simulated timber zone length distribution functions (*a* – for a trunk with a height of 30.0 m; *b* – for a point with a height of 3.0 m)

The end points of the timber wood that are shown in Fig. 1 are the points of transition of the system to the absorbing state. Calculating the

heights of these points in relative units using formula (2), it is possible to get a comparable array of experimental data (Fig. 6).

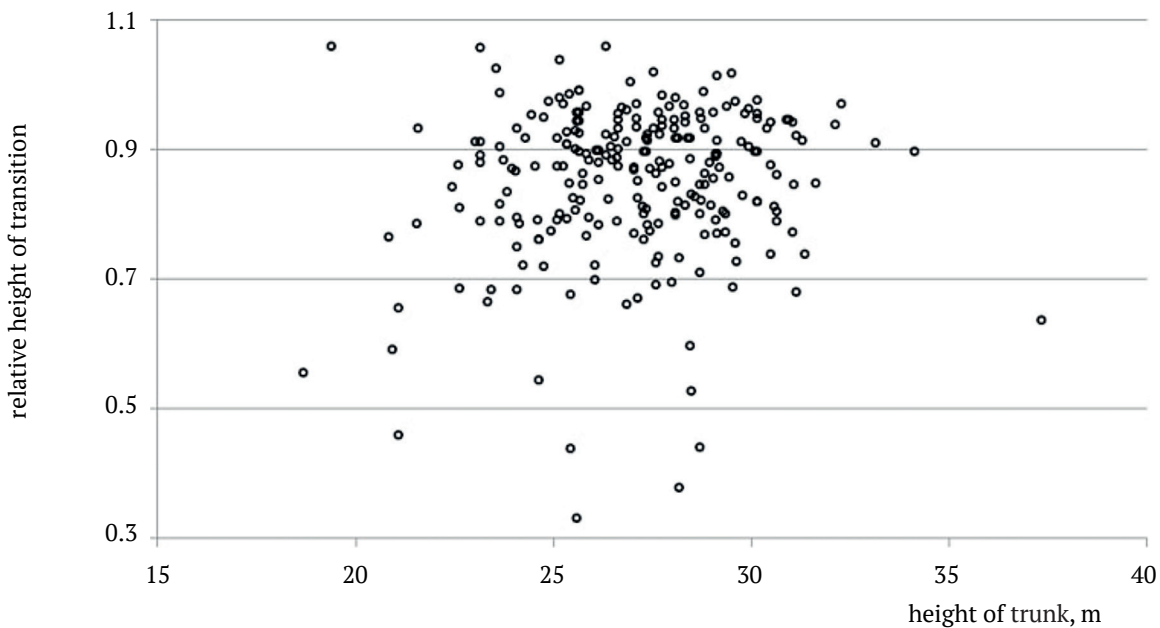


Figure 6. Dependence of the heights of the transition points to the absorbing state on the height of the trunk

As can be seen from Fig. 6, the distribution of transition points is weakly dependent on the height of the trunk. In addition, the analysed

arrays of empirical data for different groups of trunk heights by *F*- Fischer's criterion and Student's *t*-test confirmed that these groups belong

to the same general population (at the level of substantiality $\alpha=0,05$). In Figure 6 it can be seen that some values of relative heights are greater than one. These are the transition points located on the trunk in the interval $[\tilde{h}_{max}^{zone}-1; \tilde{h}_{max}^{zone}]$. When the next point hits this interval, the process will go into an absorbing state with probability 1, because the minimum length of the zone is T cannot be less than 1 m. Considering this, the transition probability distribution function can be represented as:

$$F(\tilde{h}_i) = \begin{cases} 0, & \tilde{h}_i < 0; \\ \int_0^{\tilde{h}_i} f(\tilde{h}_i) d\tilde{h}, & 0 \leq \tilde{h}_i \leq 1; \\ 1, & \tilde{h}_i > 1; \end{cases} \quad (13)$$

where $F(\hat{h}_i)$ – distribution function; $f(\hat{h}_i)$ – distribution density; \hat{h}_i – relative height.

The trunks of model trees were divided into intervals of 0.05 of the relative height (\hat{h}) to determine the distribution law and its parameters. In the future, the relative accumulated frequencies of occurrence of transition points to the state F are calculated, which fell into these gaps. Analysis of the empirical relative frequency distribution function showed that it is also approximated with sufficient accuracy by the beta distribution function (6) with the parameters: $\alpha=10,75$; $\beta=2,436$. Fig. 7 shows a graph of the distribution function (13) against the background of empirical data.

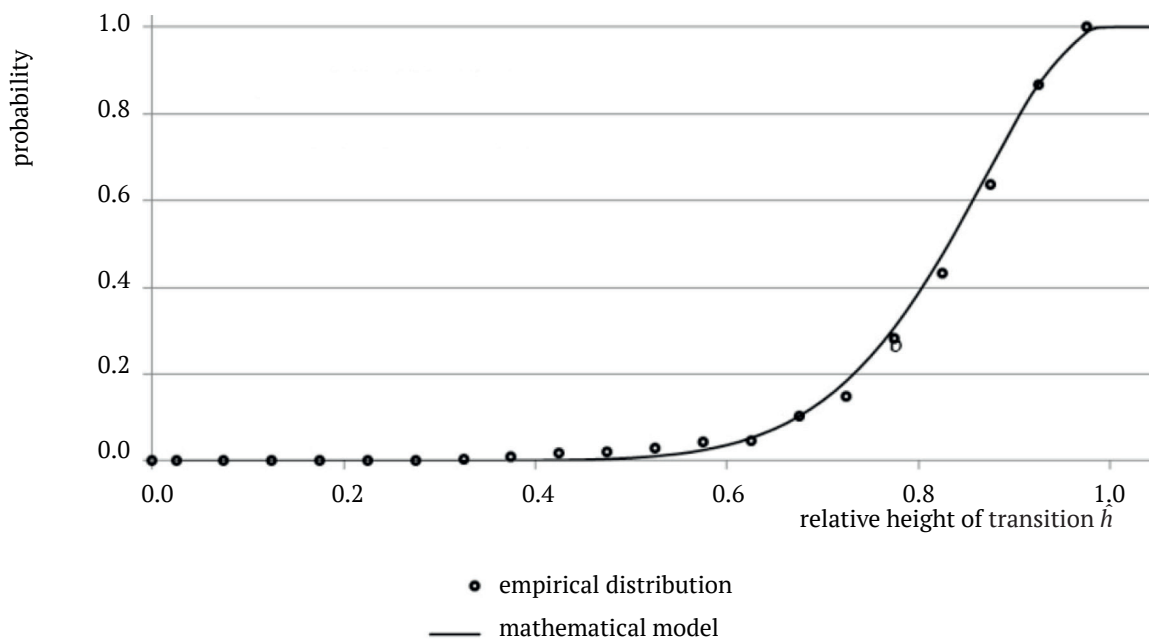


Figure 7. Distribution of relative heights of transition points to the absorbing state

Therefore, for the first row of the matrix (10) for a specific relative height, the transition probabilities can be determined using the equation (13).

The obtained mathematical models of the characteristics of a random process can be estimated using statistical methods. For the length of the zone, an interval estimate of the average is made, which is more reasonable and reliable than the point estimate. If the mathematical expectation of the zone length distribution model lies in the confidence interval of the average value for the

sample, then it is likely that the approximation error will not exceed the value of this interval.

Confidence intervals of the sample mean for probability 0.99 ($\alpha=0.01$) are calculated from the intervals of the relative height of the beginning of zones (Table 3) according to the formula:

$$\bar{x}_b \pm \frac{\sigma_b \cdot 2,58}{\sqrt{n}}, \quad (14)$$

where \bar{x}_b – sample average of the relative length of the zone; σ_b – sample standard deviation; n – number of zones.

The mathematical expectations of the beta distribution of relative zone lengths for each point in the empirical data are calculated by the formula:

$$\bar{x}_m = \frac{\alpha}{\alpha + \beta}, \quad (15)$$

where \bar{x}_m – mathematical expectation of the model; α, β – parameters of equation (6) of the Table 3. The obtained data are shown in Figure 8.

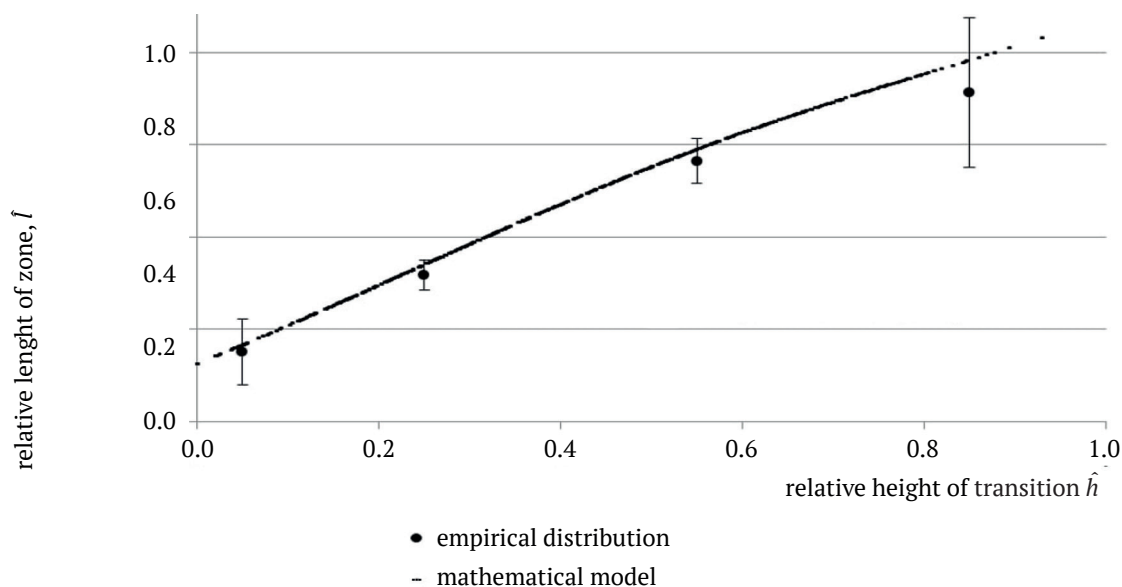


Figure 8. Confidence intervals of the average length of empirical data zones and their mathematical expectation based on the mathematical model

Analysis of the above graph shows that with a probability of 99%, the mathematical expectation of the simulated average length of the zone will be in the confidence interval, which indicates the acceptable accuracy of the mathematical model. In absolute terms, the mathematical expectation of the model is averaged over all points in the model trees (5.76 m) and approaches the empirical average (5.67 m), which also confirms this conclusion.

The accuracy of the mathematical model of the transition probability distribution (13) was acceptable both for the standard error of the simulation ($s_m=0.0184$) and the coefficient of determination ($R^2=0.996$).

Analysing the structure of the described random process, two main factors that substantially affect the final result of generating a qualitative trunk structure are distinguished.

First, it is the maximum length of timber wood. This indicator determines the scope of distribution of timber zone lengths and their possi-

ble maximum length, and the distribution of the height of the transition of timber zones to firewood ones. In practice, the height of the end of the timber part of the trunk is the basis for dividing trees into categories of technical suitability. Therefore, after modelling this indicator for semi-timber and firewood trunks, other characteristics of the random process can be easily identified.

The second factor is the form (or law) of the distribution of timber zone lengths. It is possible that for other tree species, approximating distributions will require applying different random functions than for pine, which indicates the need for further research.

The positive side of a random process is the property of creating implementations based on it that are not present in the sample but can exist with a certain probability in the general population. Evidently, if there are two identical trunks in the empirical material, in which a zone with a length of 3.0 m

and 4.0 m, respectively, begins at a certain height, then it can be argued that there are trunks in nature with a zone length of 3.1, 3.2 ... 3.9 m. Therefore, the formation of a simulation sample based on a random process considers such options, and therefore more fully reflects the object of research.

The methodology developed in this paper allows obtaining a pilot sample for further calculation of standards for the size-quality or commodity structure of plantings based on it. Admittedly, the question arises of the size of such a sample. According to previous studies (Poliakov & Poliakov, 1999, 2008, 2009), it was identified that for each “diameter-height” variant, stabilisation of the trunk volume distribution indicators occurs after 150 process generations.

Another issue is finding indicators of the trunk volume distribution based on the resulting process implementations. The most advanced methods for estimating the volume of the trunk are related to its creator, which allows for predicting the diameters from the butt to the top and calculating the median diameter and volume of any segment, knowing the height of its beginning and end. Therefore, obtaining data on the volume distribution of the array of generated trunks requires an adequate mathematical matching model and a conditional bucking algorithm in accordance with production specifications.

In recent studies, the mathematical model of the generatrix in the bark (A. Kozak, 2004), and the model of the ratio of diameters without bark and in the bark, are substantiated for ten main tree species of Ukraine. The parameters of these models were determined and tested in the framework of scientific-technical work (Developing Reference Data, 2020). The conditional bucking algorithm uses these models to determine the diameters of the generated trunk without bark at the transition points of quality zones and “bucking” the trunk so as to obtain the maximum

yield of timber of the highest quality classes. The algorithm allows using real specifications of round timber in the middle diameter without bark and length, therefore bringing the simulation result as close as possible to modern production conditions.

Conclusions

The study describes the methodological prerequisites for using a random process to model the distribution of the length of the timber part of tree trunks as the main characteristic that affects the dimensional and qualitative structure of wood reserves. The proposed approach allows predicting various combinations of biometric indicators of trees and increasing the level of reliability of mathematical models as the basis of volume tables. The results of the study show that the length of segments of the timber and firewood parts of the trunk depends primarily on the relative height of the trunk area where they are observed. For a mathematical generalisation of this process, the paper defines four zones within which the distribution of the length of the timber part of the trunk can be described by a single function. Thus, the developed algorithm allows for examining the regularities of the order of appearance of various qualitative zones of the trunk and using them during simulation modelling of the qualitative structure of tree trunks.

Despite the fact that the completed study concerns exclusively timber trunks, other patterns are possible in the distribution of quality characteristics for the trunks of semi-timber and firewood trees. In general, the results of the study indicate ways to improve the methodology for developing standards for the dimensional and qualitative structure of wood due to the lack of sufficient accumulated empirical data, which becomes especially relevant in the context of the introduction of new standards for wood classification in Ukraine.

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Моделювання якісної структури стовбурів сосни звичайної за допомогою випадкового процесу

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Анотація. У дослідженні висвітлено методика прогнозування об'єму якісних категорій стовбурової деревини для ділових стовбурів сосни звичайної, з використанням моделі розподілу різних якісних зон вздовж по стовбуру. Дослідження базується на емпіричному матеріалі, зібраному на лісосіці рубки головного користування, на якій для 245 модельних дерев визначено протяжність різних якісних зон деревини від окоренка до верхівки. Для узагальнення черговості появи та протяжності зон ділової деревини, дров і відходів у роботі використано напівмарковську ймовірнісну модель, яку визначали три параметри: 1) вхідні ймовірності появи відповідної якісної зони в окоренковій частині стовбура (початковий стан); 2) матриця розподілу довжин зони; 3) матриця ймовірностей зміни якісних зон на різних висотах стовбура. Згідно з дослідними даними, прийнято, що стовбури сосни починаються з ділової деревини. Для моделювання протяжності якісних зон стовбура застосовано функцію бета-розподілу, параметри якої підібрано залежно від відносної висоти початку відповідної зони стовбура. Ймовірності зміни якісних зон обчислено на основі емпіричних даних. У дослідженні виявлено, що розподіл довжин ділової та дров'яної деревини залежить від абсолютної висоти розташування на стовбурі та висоти стовбура. Для математичного узагальнення цього процесу в роботі визначено чотири зони, в межах яких розподіл протяжності ділової частини стовбура може описуватися єдиною функцією. Ймовірності зміни якісних зон змодельовано залежно від відносної висоти стовбурів. На цій основі опрацьовано методика імітаційного моделювання вихідних масивів даних, які можуть використовуватися з метою розроблення таблиць розмірно-якісної структури деревини для стовбурів різних діаметрів, висоти та категорій технічної придатності. Виконане дослідження стосується лише ділових стовбурів сосни, тож потрібно очікувати інші закономірності для стовбурів інших деревних видів, а також напівділових і дров'яних стовбурів. Розроблену методика доречно використовувати під час оновлення таблиць розподілу об'єму стовбурів за розмірно-якісними категоріями, які нині треба оновити шляхом запровадження нових вимог щодо класифікації ділової деревини в Україні

Ключові слова: ділова деревина, функція розподілу, напівмарковська ймовірнісна модель, таблиці розмірно-якісної структури